

## 2.8 Graphical Analysis of One Dimensional Motion

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement  $x$  vs. time  $t$  is velocity  $v$ .
- The slope of a graph of velocity  $v$  vs. time  $t$  graph is acceleration  $a$ .
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

### Conceptual Questions

#### 2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.
2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?
3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to  $50 \mu\text{m/s}$  ( $50 \times 10^{-6} \text{ m/s}$ ) have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

#### 2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "A bird that is diving for prey has a speed of  $-10 \text{ m/s}$ ." What is wrong with the student's statement? What has the student actually described? Explain.
5. What is the speed of the bird in Exercise 2.4?
6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.
7. A weather forecast states that the temperature is predicted to be  $-5^\circ\text{C}$  the following day. Is this temperature a vector or a scalar quantity? Explain.

#### 2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.
9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.
10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?
11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?
12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

#### 2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.
14. Is it possible for velocity to be constant while acceleration is not zero? Explain.
15. Give an example in which velocity is zero yet acceleration is not.
16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?
17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

#### 2.6 Problem-Solving Basics for One Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.
19. What is the last thing you should do when solving a problem? Explain.

#### 2.7 Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?
21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.
23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?
24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about  $1/6$  that of the Earth)?
25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about  $1/6$  of  $g$  on Earth)?

## 2.8 Graphical Analysis of One Dimensional Motion

26. (a) Explain how you can use the graph of position versus time in Figure 2.66 to describe the change in velocity over time. Identify (b) the time ( $t_a$ ,  $t_b$ ,  $t_c$ ,  $t_d$ , or  $t_e$ ) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

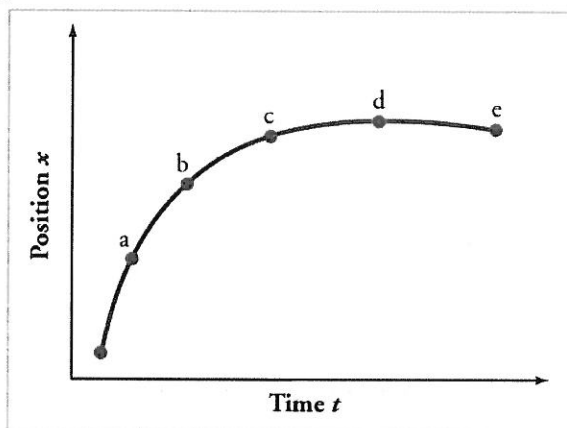


Figure 2.66

27. (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in Figure 2.67. (b) Identify the time or times ( $t_a$ ,  $t_b$ ,  $t_c$ , etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

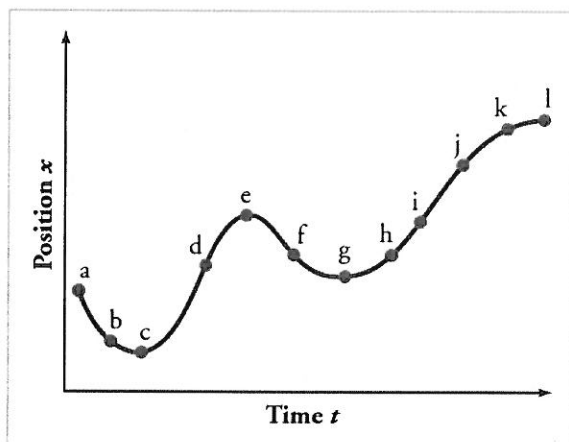


Figure 2.67

28. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 2.68. (b) Based on the graph, how does acceleration change over time?

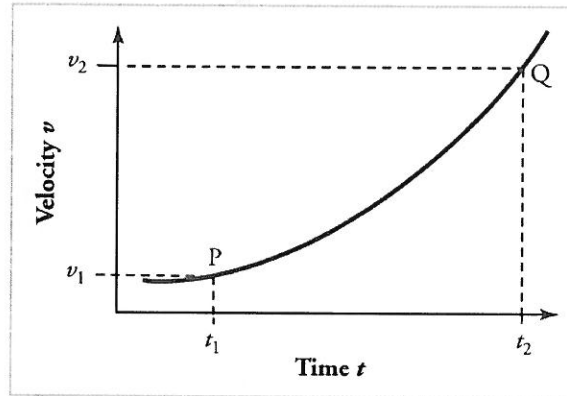


Figure 2.68

29. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 2.69. (b) Identify the time or times ( $t_a$ ,  $t_b$ ,  $t_c$ , etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

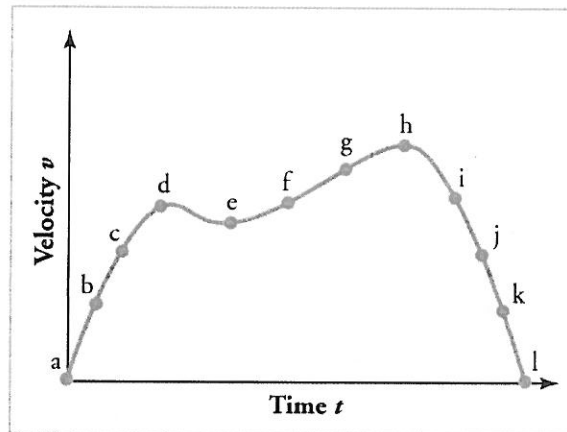


Figure 2.69

30. Consider the velocity vs. time graph of a person in an elevator shown in Figure 2.70. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from **Motion Equations for Constant Acceleration in One Dimension** for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

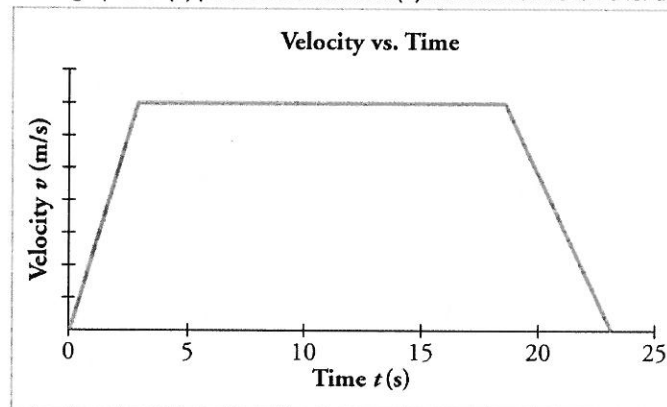


Figure 2.70

**31.** A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

## Problems & Exercises

### 2.1 Displacement

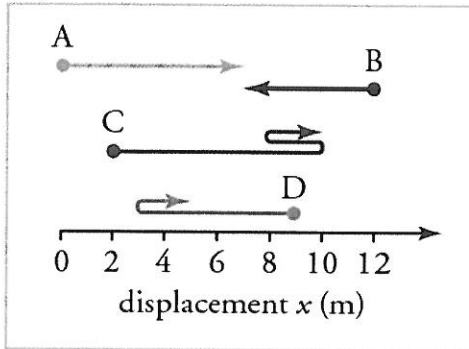


Figure 2.71

- Find the following for path A in Figure 2.71: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.
- Find the following for path B in Figure 2.71: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.
- Find the following for path C in Figure 2.71: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.
- Find the following for path D in Figure 2.71: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

### 2.3 Time, Velocity, and Speed

- (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?
- A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?
- The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?
- Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?
- On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?
- Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by  $3.84 \times 10^6$  m (1%)?

11. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction  $25.0^\circ$  south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light ( $3.00 \times 10^8$  m/s).

14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit  $1.06 \times 10^{-10}$  m in diameter. (a) If the average speed of the electron in this orbit is known to be  $2.20 \times 10^6$  m/s, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

### 2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

#### 17. Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of  $g$  ( $9.80 \text{ m/s}^2$ ) by taking its ratio to the acceleration of gravity.

18. A commuter backs her car out of her garage with an acceleration of  $1.40 \text{ m/s}^2$ . (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in  $\text{m/s}^2$  and in multiples of  $g$  ( $9.80 \text{ m/s}^2$ )?

## 2.5 Motion Equations for Constant Acceleration in One Dimension

20. An Olympic-class sprinter starts a race with an acceleration of  $4.50 \text{ m/s}^2$ . (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

21. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is  $2.10 \times 10^4 \text{ m/s}^2$ , and 1.85 ms ( $1 \text{ ms} = 10^{-3} \text{ s}$ ) elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

22. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of  $6.20 \times 10^5 \text{ m/s}^2$  for  $8.10 \times 10^{-4} \text{ s}$ . What is its muzzle velocity (that is, its final velocity)?

23. (a) A light-rail commuter train accelerates at a rate of  $1.35 \text{ m/s}^2$ . How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of  $1.65 \text{ m/s}^2$ . How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in  $\text{m/s}^2$ ?

24. While entering a freeway, a car accelerates from rest at a rate of  $2.40 \text{ m/s}^2$  for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

25. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of  $2.00 \text{ m/s}^2$ . (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

### 26. Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

27. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes  $3.33 \times 10^{-2} \text{ s}$ , calculate the distance over which the puck accelerates.

28. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

29. Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of  $0.0500 \text{ m/s}^2$  for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of  $0.550 \text{ m/s}^2$ , how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

30. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

31. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of  $0.350 \text{ m/s}^2$ , how far will it travel before becoming airborne? (b) How long does this take?

### 32. Professional Application:

A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in  $\text{m/s}^2$  and in multiples of  $g$  ( $g = 9.80 \text{ m/s}^2$ ). (b) Calculate the stopping time. (c)

The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of  $g$ ?

33. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

34. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

35. Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

36. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of  $0.150 \text{ m/s}^2$  as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

55. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

56. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms ( $8.00 \times 10^{-5}$  s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

57. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

58. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms ( $3.50 \times 10^{-3}$  s). (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

## 2.8 Graphical Analysis of One Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

59. (a) By taking the slope of the curve in Figure 2.72, verify that the velocity of the jet car is 115 m/s at  $t = 20$  s. (b) By taking the slope of the curve at any point in Figure 2.73, verify that the jet car's acceleration is  $5.0 \text{ m/s}^2$ .

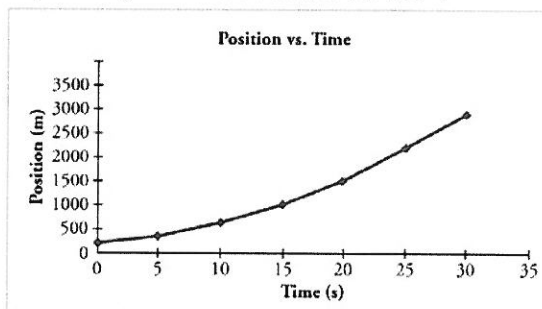


Figure 2.72

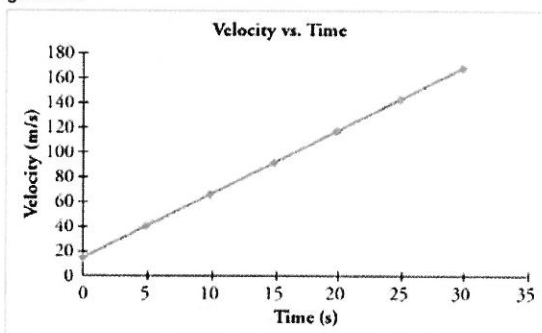


Figure 2.73

60. Using approximate values, calculate the slope of the curve in Figure 2.74 to verify that the velocity at  $t = 10.0$  s is 0.208 m/s. Assume all values are known to 3 significant figures.

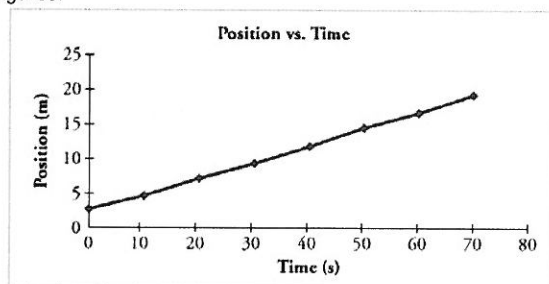


Figure 2.74

61. Using approximate values, calculate the slope of the curve in Figure 2.74 to verify that the velocity at  $t = 30.0$  s is 0.238 m/s. Assume all values are known to 3 significant figures.

62. By taking the slope of the curve in Figure 2.75, verify that the acceleration is approximately  $3.2 \text{ m/s}^2$  at  $t = 10 \text{ s}$ .

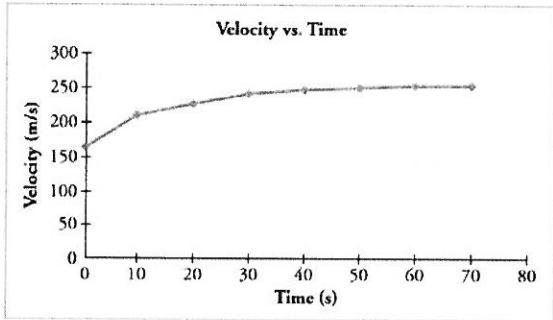


Figure 2.75

63. Construct the displacement graph for the subway shuttle train as shown in Figure 2.30(a). Your graph should show the position of the train, in kilometers, from  $t = 0$  to  $20 \text{ s}$ . You will need to use the information on acceleration and velocity given in the examples for this figure.

64. (a) Take the slope of the curve in Figure 2.76 to find the jogger's velocity at  $t = 2.5 \text{ s}$ . (b) Repeat at  $7.5 \text{ s}$ . These values must be consistent with the graph in Figure 2.77.

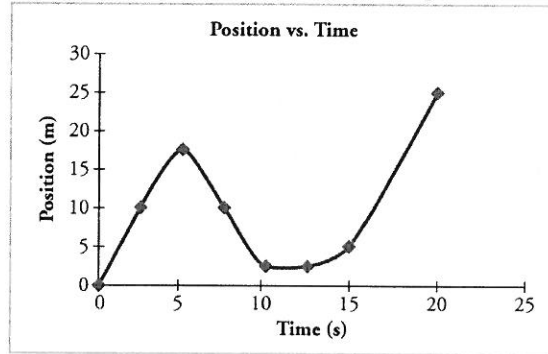


Figure 2.76

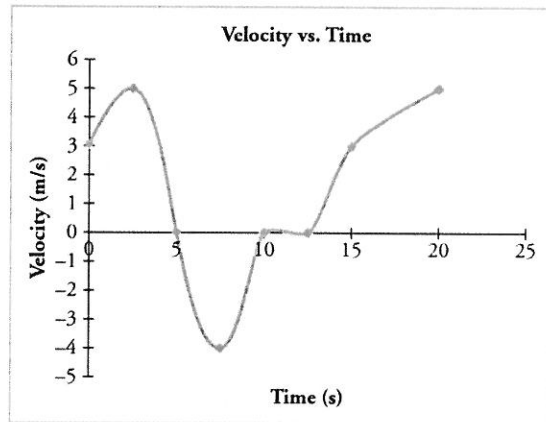


Figure 2.77

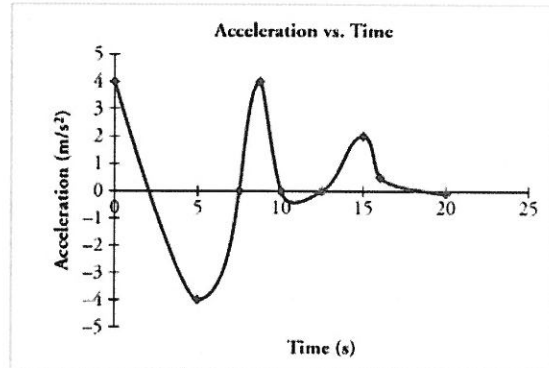


Figure 2.78



65. A graph of  $v(t)$  is shown for a world-class track sprinter in a 100-m race. (See Figure 2.79). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at  $t = 5$  s? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

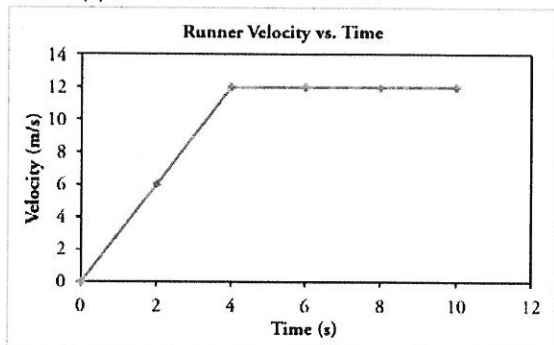


Figure 2.79

66. Figure 2.80 shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

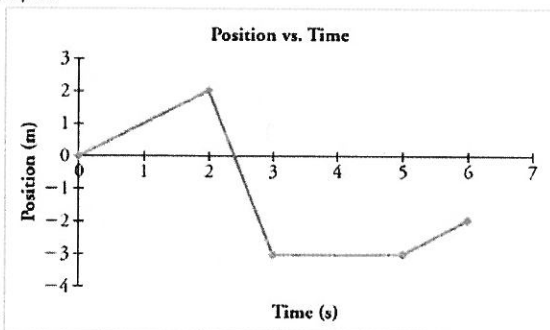


Figure 2.80

## Test Prep for AP® Courses

### 2.1 Displacement

1. Which of the following statements comparing position, distance, and displacement is correct?
- An object may record a distance of zero while recording a non-zero displacement.
  - An object may record a non-zero distance while recording a displacement of zero.
  - An object may record a non-zero distance while maintaining a position of zero.
  - An object may record a non-zero displacement while maintaining a position of zero.

### 2.2 Vectors, Scalars, and Coordinate Systems

2. A student is trying to determine the acceleration of a feather as she drops it to the ground. If the student is looking to achieve a positive velocity and positive acceleration, what is the *most sensible* way to set up her coordinate system?

- Her hand should be a coordinate of zero and the upward direction should be considered positive.
- Her hand should be a coordinate of zero and the downward direction should be considered positive.
- The floor should be a coordinate of zero and the upward direction should be considered positive.
- The floor should be a coordinate of zero and the downward direction should be considered positive.

### 2.3 Time, Velocity, and Speed

3. A group of students has two carts, *A* and *B*, with wheels that turn with negligible friction. The two carts travel along a straight horizontal track and eventually collide. Before the collision, cart *A* travels to the right and cart *B* is initially at rest. After the collision, the carts stick together.

- Describe an experimental procedure to determine the velocities of the carts before and after the collision, including all the additional equipment you would need. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.
- There will be sources of error in the measurements taken in the experiment both before and after the collision. Which velocity will be more greatly affected by this error: the velocity prior to the collision or the velocity after the collision? Or will both sets of data be affected equally? Justify your answer.

## 2.4 Acceleration

4.

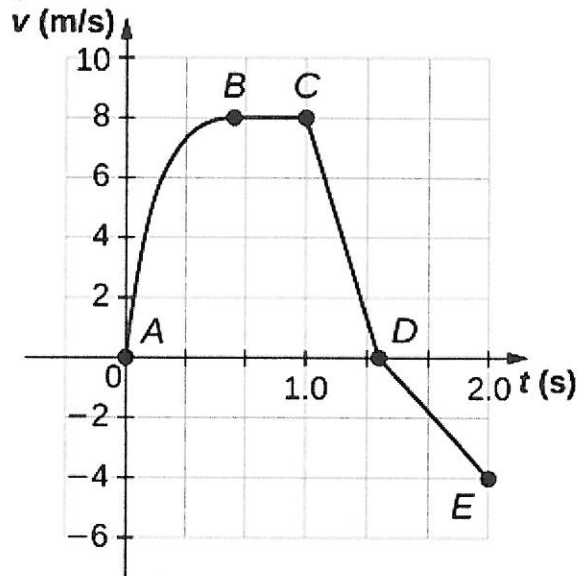


Figure 2.81 Graph showing Velocity vs. Time of a cart. A cart is constrained to move along a straight line. A varying net force along the direction of motion is exerted on the cart. The cart's velocity  $v$  as a function of time  $t$  is shown in the graph. The five labeled points divide the graph into four sections.

Which of the following correctly ranks the magnitude of the average acceleration of the cart during the four sections of the graph?

- $a_{CD} > a_{AB} > a_{BC} > a_{DE}$
- $a_{BC} > a_{AB} > a_{CD} > a_{DE}$
- $a_{AB} > a_{BC} > a_{DE} > a_{CD}$
- $a_{CD} > a_{AB} > a_{DE} > a_{BC}$

5. Push a book across a table and observe it slow to a stop.

Draw graphs showing the book's position vs. time and velocity vs. time if the direction of its motion is considered positive.

Draw graphs showing the book's position vs. time and velocity vs. time if the direction of its motion is considered negative.

## 2.5 Motion Equations for Constant Acceleration in One Dimension

6. A group of students is attempting to determine the average acceleration of a marble released from the top of a long ramp. Below is a set of data representing the marble's position with

respect to time.

Position (cm)	Time (s)
0.0	0.0
0.3	0.5
1.25	1.0
2.8	1.5
5.0	2.0
7.75	2.5
11.3	3.0

Use the data table above to construct a graph determining the acceleration of the marble. Select a set of data points from the table and plot those points on the graph. Fill in the blank column in the table for any quantities you graph other than the given data. Label the axes and indicate the scale for each. Draw a best-fit line or curve through your data points.

Using the best-fit line, determine the value of the marble's acceleration.

## 2.7 Falling Objects

7. Observing a spacecraft land on a distant asteroid, scientists notice that the craft is falling at a rate of 5 m/s. When it is 100 m closer to the surface of the asteroid, the craft reports a velocity of 8 m/s. According to their data, what is the approximate gravitational acceleration on this asteroid?

- 0 m/s<sup>2</sup>
- 0.03 m/s<sup>2</sup>
- 0.20 m/s<sup>2</sup>
- 0.65 m/s<sup>2</sup>
- 33 m/s<sup>2</sup>